

NAG Toolbox for MATLAB

f07gj

1 Purpose

f07gj computes the inverse of a real symmetric positive-definite matrix A , where A has been factorized by f07gd, using packed storage.

2 Syntax

```
[ap, info] = f07gj(uplo, n, ap)
```

3 Description

f07gj is used to compute the inverse of a real symmetric positive-definite matrix A , the function must be preceded by a call to f07gd, which computes the Cholesky factorization of A , using packed storage.

If **uplo** = 'U', $A = U^T U$ and A^{-1} is computed by first inverting U and then forming $(U^{-1})U^{-T}$.

If **uplo** = 'L', $A = LL^T$ and A^{-1} is computed by first inverting L and then forming $L^{-T}(L^{-1})$.

4 References

Du Croz J J and Higham N J 1992 Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates how A has been factorized.

uplo = 'U'

$A = U^T U$, where U is upper triangular.

uplo = 'L'

$A = LL^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – int32 scalar

n , the order of the matrix A .

Constraint: $n \geq 0$.

3: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The Cholesky factor of A stored in packed form, as returned by f07gd.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.

The factorization contains the n by n matrix A^{-1} .

More precisely,

if **uplo** = 'U', the upper triangle of $\{A\}^{-1}$ must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;
 if **uplo** = 'L', the lower triangle of $\{A\}^{-1}$ must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **info**.

info > 0

If **info** = i , the i th diagonal element of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \quad \text{and} \quad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n , ϵ is the *machine precision* and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2 \|A^{-1}\|_2.$$

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogue of this function is f07gw.

9 Example

```
uplo = 'L';
n = int32(4);
ap = [2.039607805437114;
      -1.529705854077835;
      0.2745625891934577;
```

```
-0.04902903378454601;  
1.640121946685673;  
-0.2499814119483738;  
0.6737303907389101;  
0.7887488055748053;  
0.6616575633742563;  
0.5346894269298685];  
[apOut, info] = f07gj(uplo, n, ap)
```

```
apOut =  
    0.6995  
    0.7769  
    0.7508  
   -0.9340  
    1.4239  
    1.8255  
   -1.8841  
    4.0688  
   -2.9342  
    3.4978  
info =  
      0
```
